# Technical Notes

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## A Model for Vortex Breakdown on Slender Wings

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### Introduction

THE research reported herein deals with a new theoretical model for evaluating the effect of vortex breakdown on the aerodynamics of slender wings. As is well known, vortex breakdown affects the aerodynamic performance of a lifting surface by decreasing the peak pressure and, consequently, the wing's lift. In the present method, the vortex breakdown phenomenon is represented by a distribution of sources. This model is a suitable idealization of the phenomenon, considering that the main effect of the breakdown on the external stream is similar to that of a suddenly expanding streamtube, as illustrated in Fig. 1. The effect of vortex breakdown on the aerodynamics of wings has been considered in previous works[1] as a correction based on the leading-edge suction analogy or empirical information. The drawback of such approaches is that they are not capable of predicting the flowfield in the vicinity of the wing. The present method is free of this limitation.

### Theory

The model is applicable to simple, slender wings with two concentrated vortices. The main assumptions are that the wing, together with the vortex structure, is a slender system, and that the vortex breakdown is of the bubble type, as far as its influence on the far field is concerned. The last assumption asserts that the breakdown type does not have a primary effect on the wing aerodynamic properties.

The present work is essentially a feasibility study of the wing aerodynamics including vortex breakdown, not dealing with the breakdown mechanism. Therefore, a relatively simple model for the vortical flow has been assumed. The chosen basic model is the extension of the Brown and Michael[2] model proposed by Smith,[3] shown in Fig. 1. A conical solution is assumed near the wing's apex and is used as an initial condition for the general solution. Downstream of the breakdown location, a distribution of two-dimensional sources along the vortex axis is assumed. This generates a semi-infinite slender body. As a result of the slenderness assumption, there exists in each crossflow plane a velocity potential that satisfies the two-dimensional Laplace equation.

The boundary condition at the wing's surface which has semi-span $a$ is

$$Im\left\{\frac{\partial W}{\partial \sigma}\right\}_{Z=0,\,|Y|<a} = 0$$

and the condition at infinity is

$$\lim_{|\sigma|\to\infty}\{W\} = -iV\sin\alpha\cdot\sigma + 2\cdot\frac{Q}{2\pi}\,\ell n\,\sigma$$

when the complex potential representation in the $\sigma$ plane is

$$W = \frac{-i\Gamma}{2\pi}\,\ell n\left(\frac{\sqrt{\sigma^2-a^2}-\sqrt{\sigma_0^2-a^2}}{\sqrt{\sigma^2-a^2}+\sqrt{\bar{\sigma}_0^2-a^2}}\right) + \frac{Q}{2\pi}\,\ell n(\sqrt{\sigma^2-a^2}$$
$$-\sqrt{\sigma_0^2-a^2})(\sqrt{\sigma^2-a^2}+\sqrt{\bar{\sigma}_0^2-a^2}) - iV\sin\alpha\cdot\sqrt{\sigma^2-a^2} \qquad (1)$$

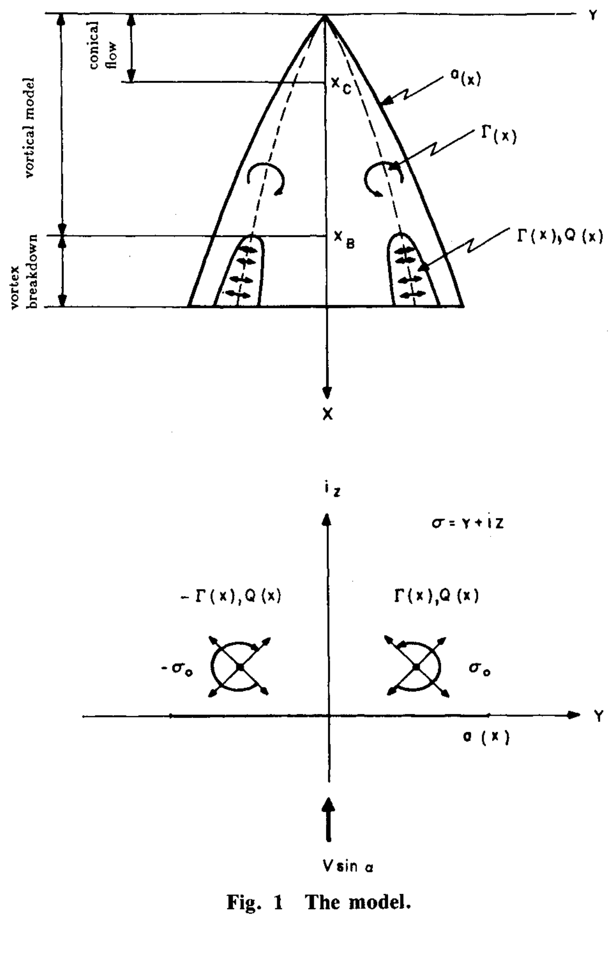


Fig. 1 The model.

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and $\Gamma, Q$ are the vortex and source strengths respectively. From the Kutta condition (finite velocity at the leading edge), the dependence of the circulation on the angle of attack and the assumed source strength is expressed as

$$\Gamma = \frac{2\pi V \sin\alpha - iQ(1/\sqrt{\sigma_0^2 - a^2} - 1/\sqrt{\bar{\sigma}_0^2 - a^2})}{1/\sqrt{\sigma_0^2 - a^2} + 1/\sqrt{\bar{\sigma}_0^2 - a^2}} \tag{2}$$

It can be seen that the term

$$i(1/\sqrt{\sigma_0^2 - a^2} - 1/\sqrt{\bar{\sigma}_0^2 - a^2})$$

is real and positive, and consequently sources cause a reduction of circulation.

The last condition requires that the resultant force on the vortex source and the feeding sheet be zero. (It should be noted that the forces on the vortex and source are perpendicular to each other.) From this consideration, the following differential equation is obtained:

$$\begin{aligned}
&V\left\{\left(\frac{\Gamma - iQ}{\Gamma^2 + Q^2}\right)(\bar{\sigma}_0 - a)\cdot\lambda_1 + \cos\alpha\right\}\cdot\frac{\mathrm{d}y_0}{\mathrm{d}x} \\
&+ V\left\{\left(\frac{\Gamma - iQ}{\Gamma^2 + Q^2}\right)(\bar{\sigma}_0 - a)\cdot\lambda_2 - i\cos\alpha\right\}\cdot\frac{\mathrm{d}z_0}{\mathrm{d}x} \\
&+ V\left(\frac{\Gamma - iQ}{\Gamma^2 + Q^2}\right)(\bar{\sigma}_0 - a)\left(\lambda_3\frac{\mathrm{d}a}{\mathrm{d}x} + \lambda_4\frac{\mathrm{d}Q}{\mathrm{d}x}\right) \\
&- i\frac{\Gamma}{2\pi}\left\{\frac{a^2}{2\sigma_0(\sigma_0^2 - a^2)} + \frac{\sigma_0}{(\sigma_0^2 - a^2) + \sqrt{(\sigma_0^2 - a^2)(\bar{\sigma}_0^2 - a^2)}}\right. \\
&\left. - \frac{\sigma_0}{(\sigma_0^2 - a^2)} - \frac{\sigma_0}{\sqrt{(\sigma_0^2 - a^2)(\bar{\sigma}_0^2 - a^2)}}\right\} \\
&+ \frac{Q}{2\pi}\left\{\frac{a^2}{2\sigma_0(\sigma_0^2 - a^2)} - \frac{\sigma_0}{(\sigma_0^2 - a^2)\sqrt{(\sigma_0^2 - a^2)(\bar{\sigma}_0^2 - a^2)}}\right. \\
&\left. - \frac{\sigma_0}{(\sigma_0^2 - a^2)} + \frac{\sigma_0}{\sqrt{(\sigma_0^2 - a^2)(\bar{\sigma}_0^2 - a^2)}}\right\} = 0
\end{aligned} \tag{3}$$

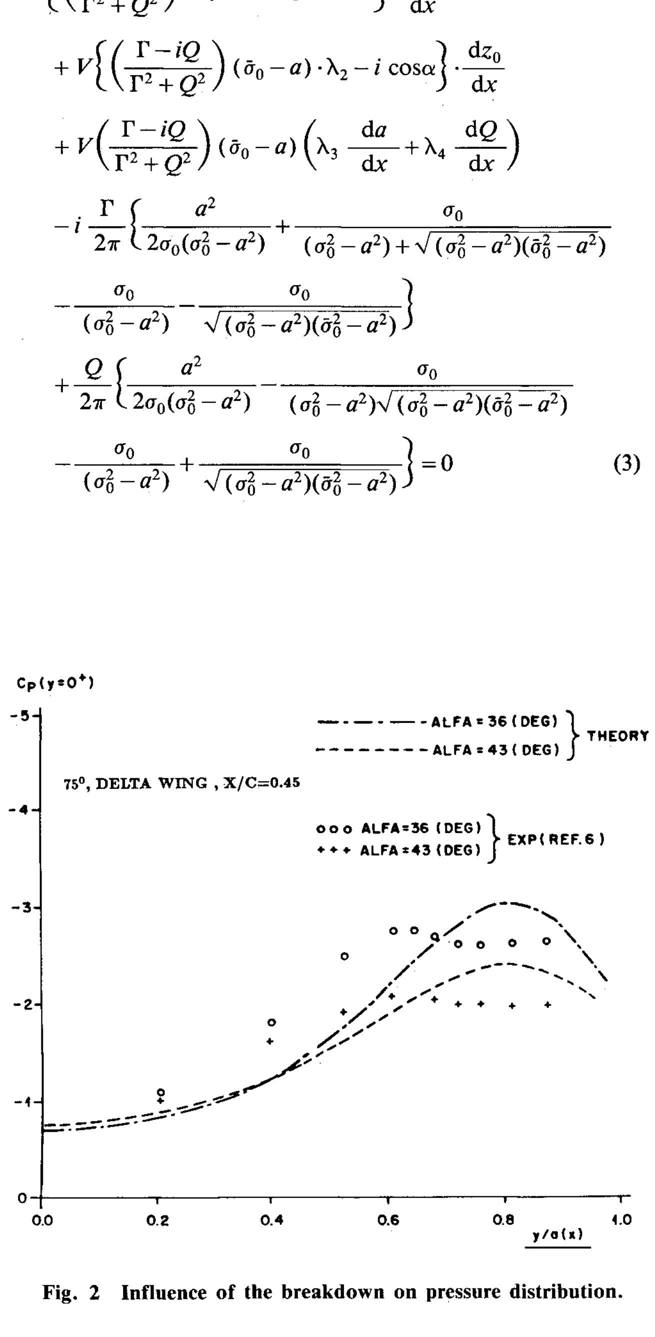


**Fig. 2 Influence of the breakdown on pressure distribution.**

where $\lambda_1$, $\lambda_2$, $\lambda_3$, and $\lambda_4$ are real functions

$$\begin{aligned}
\lambda_1 = &\frac{2\pi V\sin\alpha + 2iQ(1/\sqrt{\bar{\sigma}_0^2 - a^2})}{(\sigma_0^2 - a^2)(1/\sqrt{\sigma_0^2 - a^2} + 1/\sqrt{\bar{\sigma}_0^2 - a^2})^2}\cdot\frac{\sigma_0}{\sqrt{\sigma_0^2 - a^2}} \\
&+ \frac{2\pi V\sin\alpha - 2iQ(1/\sqrt{\sigma_0^2 - a^2})}{(\bar{\sigma}_0^2 - a^2)(1/\sqrt{\sigma_0^2 - a^2} + 1/\sqrt{\bar{\sigma}_0^2 - a^2})^2}\cdot\frac{\bar{\sigma}_0}{\sqrt{\bar{\sigma}_0^2 - a^2}} \\
\lambda_2 = i&\left\{\frac{2\pi V\sin\alpha + 2iQ(1/\sqrt{\bar{\sigma}_0^2 - a^2})}{(\sigma_0^2 - a^2)(1/\sqrt{\sigma_0^2 - a^2} + 1/\sqrt{\bar{\sigma}_0^2 - a^2})^2}\cdot\frac{\sigma_0}{\sqrt{\sigma_0^2 - a^2}}\right. \\
&\left. - \frac{2\pi V\sin\alpha - 2iQ(1/\sqrt{\sigma_0^2 - a^2})}{(\bar{\sigma}_0^2 - a^2)(1/\sqrt{\sigma_0^2 - a^2} + 1/\sqrt{\bar{\sigma}_0^2 - a^2})^2}\cdot\frac{\bar{\sigma}_0}{\sqrt{\bar{\sigma}_0^2 - a^2}}\right\} \\
\lambda_3 = &-\frac{2\pi V\sin\alpha + 2iQ(1/\sqrt{\bar{\sigma}_0^2 - a^2})}{(\sigma_0^2 - a^2)(1/\sqrt{\sigma_0^2 - a^2} + 1/\sqrt{\bar{\sigma}_0^2 - a^2})^2}\cdot\frac{\sigma_0}{\sqrt{\sigma_0^2 - a^2}} \\
&- \frac{2\pi V\sin\alpha - 2iQ(1/\sqrt{\sigma_0^2 - a^2})}{(\bar{\sigma}_0^2 - a^2)(1/\sqrt{\sigma_0^2 - a^2} + 1/\sqrt{\bar{\sigma}_0^2 - a^2})^2}\cdot\frac{\bar{\sigma}_0}{\sqrt{\bar{\sigma}_0^2 - a^2}} \\
\lambda_4 = &-i\left(\frac{1/\sqrt{\sigma_0^2 - a^2} - 1/\sqrt{\bar{\sigma}_0^2 - a^2}}{1/\sqrt{\sigma_0^2 - a^2} + 1/\sqrt{\bar{\sigma}_0^2 - a^2}}\right)
\end{aligned}$$

These functions were obtained from the expression for the rate of shedding of circulation

$$\frac{\mathrm{d}\Gamma}{\mathrm{d}x} = \lambda_1\cdot\frac{\mathrm{d}Y_0}{\mathrm{d}x} + \lambda_2\cdot\frac{\mathrm{d}Z_0}{\mathrm{d}x} + \lambda_3\cdot\frac{\mathrm{d}a}{\mathrm{d}x} + \lambda_4\cdot\frac{\mathrm{d}Q}{\mathrm{d}x}$$

(It can be seen that $\lambda_4$ is real and negative, and consequently the expanding bubble causes a reduction of the vorticity shedding.) The solution of the differential Eq. (3) together with Eq. (2) and the source strength distribution as input gives the vortex trajectory.

## Results

In the present work, the source strength is an external input. The determination of the optimum source distribution for the purpose of vortex breakdown representation is beyond the scope of this work. Therefore, an engineering approximation for estimating the source strength has been adopted for checking the model. The flow visualization of Ref. 4 indicates that in most regions downstream of vortex breakdown on delta wings, a linear relation between the bubble diameter and the wing's semispan is a good approximation (when the breakdown occurs at the apex, most of the wing is covered by the bubble).

Various shapes of the bubble's leading edge were tried and were found to have a negligible effect on the wing's aerodynamic parameters. Therefore the simplest conical geometry was adopted. As previously mentioned, the source strength determination is under the assumption of a bubble-type vortex breakdown analogous to a similar slender body. The location of the breakdown has been taken from Ref. 5. The lift and pressure distributions of several delta wings have been compared with the experimental results of Refs. 5 and 6.

In Fig. 2 there is a good agreement in the decrease of peak pressure due to the breakdown. However, it is seen that the predicted lateral location of the pressure peak is further outboard than the measured one. This inaccuracy is to be expected because of the vortical model's simplicity.

Figure 3 describes the lift coefficient of wings with sweep angles of 75 and 80 degs. The present method overestimated lift, but the influence of the breakdown on lift is qualitatively in good agreement with the measured results, especially for the lift curve slope.

In summary, it appears that sources can be successfully used to represent the influence of vortex breakdown on wings. The use of more accurate models, e.g., those having

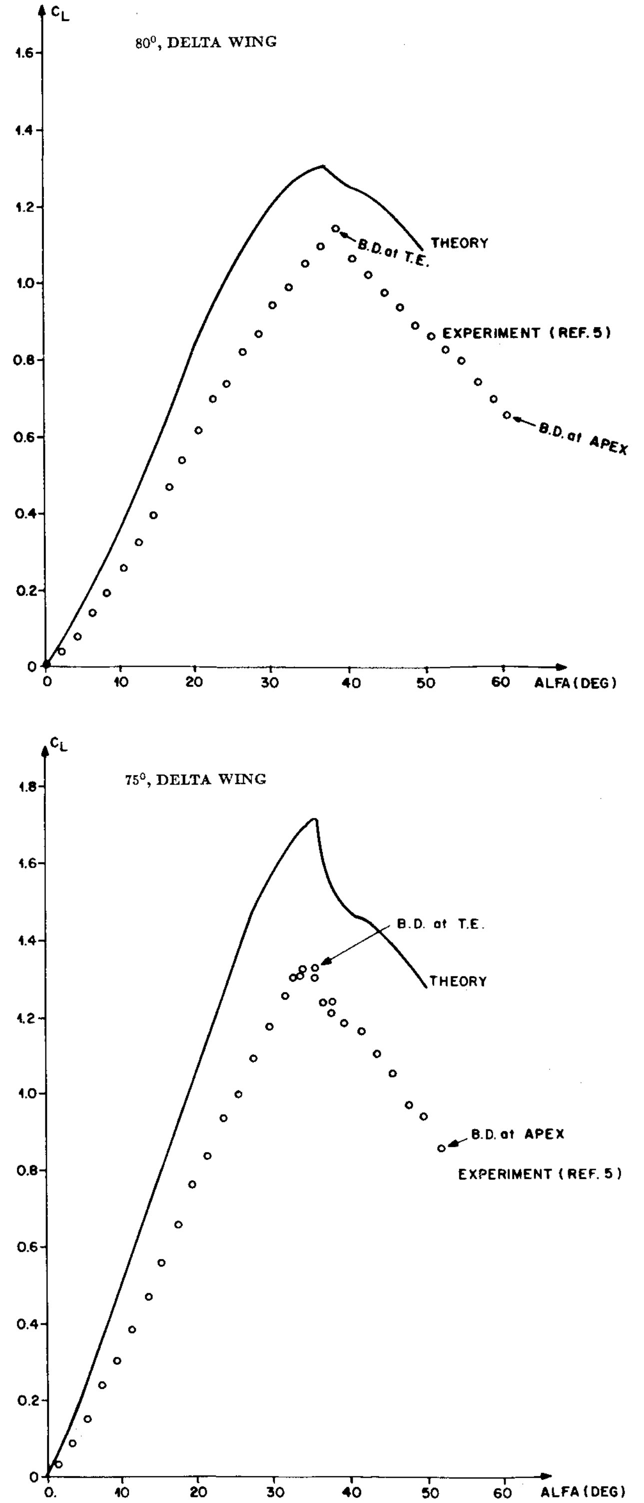


**Fig. 3 Influence of the breakdown on lift coefficient.**

three-dimensional characteristics, should improve the agreement between the present method and the experimental results.

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# Baldwin-Lomax Factors for Turbulent Boundary Layers in Pressure Gradients

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## Introduction

THE Baldwin-Lomax relations[1] for eddy viscosity have become advantageous for solving the Reynolds-averaged Navier-Stokes equations for turbulent shear flows.[2,3] Baldwin and Lomax modified the Cebeci-Smith algebraic model by substituting new relations for conditions at the outer edge of the shear flow. The numerical solution was facilitated by eliminating the need to determine the displacement thickness and the boundary-layer thickness. To accomplish this, two factors, $C_{\text{kleb}}$ and $C_{cp}$, were introduced and given constant values. York and Knight[4] used other constant values for $C_{\text{kleb}}$ and $C_{cp}$ when applying the Baldwin-Lomax model to nonseparating, incompressible (low-Mach number) turbulent boundary layers in pressure gradients. Even when using constant values of $C_{\text{kleb}}$ and $C_{cp}$, York and Knight indicated a variation with the Coles wake factor. However, the extent of the variation was not given. Accordingly, this technical note derives relations that show a large variation of Baldwin-Lomax factors $C_{\text{kleb}}$ and $C_{cp}$ with the Coles wake factor representing favorable pressure gradients as well as adverse pressure gradients up to separation. The basis is the well-substantiated outer similarity velocity law. By eliminating the Coles wake factor a formula is derived for $C_{cp}$ as a function of $C_{\text{kleb}}$. For the case of equilibrium boundary layers, the Baldwin-Lomax factors are given as functions of a modified Clauser pressure-gradient parameter. Finally, the variation of the Clauser factor ($k$) with low Reynolds number is adapted to the Baldwin-Lomax model.

## Baldwin-Lomax Model

For the outer region, which may encompass most of the boundary layer, the kinematic eddy viscosity ($\nu_t$) in the Baldwin-Lomax model may be written, in their rather odd notation, for turbulent boundary layers as

$$\nu_t = kC_{cp}F_{\text{wake}}[1+5.5(C_{\text{kleb}}y/y_{\max})^6]^{-1} \tag{1}$$

$$F_{\text{wake}} = y_{\max}F_{\max} \tag{2}$$

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